Approximated Properties of the Modified Periodogram for Stationary Time Series of Two Vector Valued With Missed Data

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Abstract -The asymptotic properties of the periodogram for stationary two vector valued time series with missed data is presented, and the dispersion properties are investigated.

Keywords: Data Window, Discrete Time Series, Modified Periodogram, Stability Process, Time Series.

1 Introduction

This paper is interested to investigate the asymptotic moments of the modified Periodogram based on Data window properties. Many authors as, D.R. Brillinger, [1], EA Farag, MA Ghazal [2], [3], M. A. Ghazal, G.S. Mokaddis, and A. El-Desokey [4], M. A. Ghazal, E. A. El-Desokey,., and Alargt, M.A [5], [6], Ghazal, M. A., A. I., El-Desokey, and A. M. Ben Aros [7], studied the statistical analysis of function of time series with missed observations of discrete and continuous cases. The paper is organized as follow, Section 1, Introduction, Section 2 we will study the approximated properties of the modified periodogram for two vector valued stationary time series with missed observations, section 3 will study the dispersion of the modified periodogram for two vector valued stationary time series with missed observations.

2 The Asymptotic Properties of The Modified Periodogram For Two Vector Valued Stationary Time Series With Missed Observations

Let an $(i+j)$ two vector-valued stationary time series

$$\tau(t)=\left[X(t) \quad \psi(t)\right]^T \tag{2.1}$$

$t=0,\pm1,\pm2,\ldots$ with $X(t)$, i- vector-valued and $\psi(t)$, j- vector-valued. Let the series (2.1) is a strictly stationary $(i+j)$ vector-valued time series with $\left[X_s(t) \quad \psi_r(t)\right]^T$, $s=1,2,\ldots,i$, $r=1,2,\ldots\ldots,j$, with existing moments as follows,

$$EX(t)=R_x \ , \ E\psi(t)=R_\psi \ , \tag{2.2}$$

and the covariances

$$E\left\{[X(t+v)-R_x][X(t)-R_x]^T\right\}=R_{xx}(v)$$

$$E\left\{[X(t+v)-R_x][\psi(t)-R_\psi]^T\right\}=R_{x\psi}(v)\,, \tag{2.3}$$

$$E\left\{[\psi(t+v)-R_\psi][\psi(t)-R_\psi]^T\right\}=R_{\psi\psi}(v)\,,$$

the spectral densities are defined as,

$$f_{xx}(h)=(2\pi)^{-1}\sum_{v=-\infty}^{\infty}R_{xx}(v)\,Exp(-ihv)\,,$$

$$f_{x\psi}(h)=(2\pi)^{-1}\sum_{v=-\infty}^{\infty}R_{x\psi}(v)\,Exp(-ihv)\,, \tag{2.4}$$

$$f_{\psi\psi}(h)=(2\pi)^{-1}\sum_{v=-\infty}^{\infty}R_{\psi\psi}(v)\,Exp(-ihv)\,,$$

$$for -\infty<h<\infty .$$

From the previous we consider the following Assumption,

Assumption I. *Let* $\lambda_a^{(T)}(t), t\in R$, $a=\overline{1,i}$ *be bounded, and vanishes for* $t>T-1, t<0$, *is called data window. Then consider*

$$\gamma_{a_1,\ldots,a_k(h)}=\sum_{t=0}^{T-1}\left[\prod_{s=1}^{k}\lambda_{a_s}^{(T)}(t)\right]Exp\{-iht\},$$

for $-\infty<h<\infty$, *and* $a_1,\ldots, a_k = 1,2,\ldots, i$.

Consider,

$d_a^{(T)}(h)$ *is the discrete expanded finite Fourier transform which is defined by*

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$$d_a^{(T)}(h)=\left[2\pi\sum_{t=0}^{T-1}\left(\lambda_a^{(T)}(t)\right)^2\right]^{-1/2}C_{\alpha\alpha}^{(T)}(h),-\infty<h<\infty, \quad (2.5)$$

where,

$$C_{\alpha\alpha}^{(T)}(h)=\begin{bmatrix}C_{X_a}^{(T)}(h)\\ C_{\psi_a}^{(T)}(h)\end{bmatrix}=\sum_{t=0}^{T-1}\lambda_a^{(T)}(t)\alpha_a^{(T)}(t)Exp\{-ihv\}, (2.6)$$

$$\alpha_a(t)=\ell_a(t)\tau_a(t)\ ,a=1,2,......,\min(i,j), \quad (2.7)$$

Where, $\ell_a(t)$ *is a Bernoulli sequence of random variable and satisfies*

$$\ell_a(t)=\begin{cases}1 & , if\ X_a(t),\Psi_a(t)\ are\ observed\ ;\\ 0 & ,\ o.w.\end{cases} \quad (2.8)$$

and $\ell_a(t)$ *satisfies*

$$P[\ell_a(t)=1]=p_a\ ,$$
$$P[\ell_a(t)=0]=q_a\ , \quad (2.9)$$

where, $p_a+q_a=1$, *and* $\lambda_a^{(T)}(t)=\lambda_a^{(T)}(\frac{t}{T})$, $t\in[0,T]$ *has bounded variation and vanishes for all* t *outside* $[0,T]$, *which is called data window function.*

Theorem 2.1. *Let* $\alpha_a(t)=\ell_a(t)\tau_a(t)$, $a=1,2,...,min\ (i,j)$ *are missed observations on the stationary stochastic processes* $X_a(t),\psi_a(t)\ ,a=1,2,..,min(i,j)$ *and* $\ell_a(t)$ *is Bernoulli sequence of random variables which satisfies (2.8), (2.9), then*

$$E\{\alpha_a(t)\}=0 \quad (2.10)$$

$$Cov\{\alpha_{a_1}(t_1),\alpha_{a_2}(t_2)\}=p_{a_1a_2}\begin{bmatrix}R_{xx}(v) & R_{x\psi}(v)\\ R_{\psi x}(v) & B(\Omega)R_{xx}(v)B(\Omega)^T\end{bmatrix}, \quad (2.11)$$

Proof. The proof is omitted.

Assumption II. *Let* $X(t)$ *is a strictly stationary time series whose moments exist. For each* $s=1,2,...,k-1$ *and any k-tuple* $a_1,a_2,......,a_k$ *we have*

$$\sum_{v_1,....,v_{k-1}}\left|v_s\ R_{a_1,....,a_k}(v_1,....,v_{k-1})\right|<\infty\ ,\ k=2,3,... ,$$

where,

$R_{a_1,....,a_k}(v_1,....,v_{k-1})=cum\{X_{a_1}(t+v_1),X_{a_1}(t+v_2),....,X_{a_k}(t)$, $(a_1,....,a_k=1,2,....,i,v_1,....,v_{k-1},t\in R,k=2,3,...)$ see [1].

Theorem 2.2. *Let* $\alpha_a(t)=\ell_a(t)\tau_a(t)$, $a=1,2,...,min\ (i,j)$ *are missed observations on the stationary stochastic processes* $X_a(t),\psi_a(t)\ ,a=1,2,...,\min(i,j)$ *and* $\ell_a(t)$ *is Bernoulli sequence of random variables which satisfies assumption II with mean zero, and let* $\lambda_a(t),-\infty<t<\infty$, *satisfies Assumption II for* $a=1,...,min(i,j)$, *and let*

$$I_{\alpha\alpha}^{(T)}(h)=\left[I_{ab}^{(T)}(h)\right]=\left[\left\{2\pi\gamma_{ab}^{(T)}(0)\right\}^{-1}C_a^{(T)}(h)\overline{C_b^{(T)}(h)}\right], (2.12)$$

where the bar denotes the complex conjugate. Then

$$E\left[I_{ab}^{(T)}(h)\right]=P_{ab}\begin{bmatrix}f_{a_1a_2}(h) & f_{a_1b_2}(h)\\ f_{b_1a_2}(h) & B(h)f_{a_1a_2}(h)B(h)^T\end{bmatrix}+$$
$$+\begin{bmatrix}O(T^{-1}) & O(T^{-1})\\ O(T^{-1}) & O(T^{-1})\end{bmatrix}, \quad (2.13)$$

Where, $O(T^{-1})$ *is uniform in* h.

$$Cov\left[I_{a_1b_1}^{(T)}(h),I_{a_2b_2}^{(T)}(\hbar)\right]=\left\{\gamma_{a_1b_1}^{(T)}(0)\gamma_{a_2b_2}^{(T)}(0)\right\}^{-1}\times$$
$$\left[P^4\gamma_{a_1b_2}(h-\hbar)\overline{\gamma_{b_1a_2}(h-\hbar)}\eta A\right], \quad (2.14)$$
$$+P^4\gamma_{a_1b_2}(h+\hbar)\overline{\gamma_{b_1a_2}(h+\hbar)}\eta A\Big]+T^{-2}N_{a_1b_1a_2b_2}^{(T)}(h,\hbar)$$
$$+O(T^{-1}),$$

where,

$$\eta=\begin{bmatrix}f_{a_1a_2}(h) & f_{a_1b_2}(h)\\ f_{b_1a_2}(h) & B(h)f_{a_1a_2}(h)B(h)^T\end{bmatrix},$$

$$A=\begin{bmatrix}f_{a_1a_2}(-h) & f_{a_1b_2}(-h)\\ f_{b_1a_2}(-h) & B(h)f_{a_1a_2}(-h)B(h)^T\end{bmatrix}$$

Proof.

$$I_{\alpha\alpha}^{(T)}(h)=\left[I_{ab}^{(T)}(h)\right]=\left[\left\{2\pi\gamma_{ab}^{(T)}(0)\right\}^{-1}C_a^{(T)}(h)\overline{C_b^{(T)}(h)}\right] ,$$

$$E\left[I_{ab}^{(T)}(h)\right]=\left\{2\pi\gamma_{ab}^{(T)}(0)\right\}^{-1}\times$$
$$\times\sum_{t_1,t_2=0}^{T-1}\lambda_a^{(T)}(t_1)\lambda_b^{(T)}(t_2)\exp(-iht_1+iht_2)E\left[\alpha_a(t_1)\alpha_b(t_2)\right]$$
$$=\left\{2\pi\gamma_{ab}^{(T)}(0)\right\}^{-1}\times\sum_{t_1,t_2=0}^{T-1}\lambda_a^{(T)}(t_1)\lambda_b^{(T)}(t_2)\exp(-iht_1+iht_2)\times$$
$$\times\left[Cov(\alpha_a(t_1),\alpha_b(t_2))+E(\alpha_a(t_1))E(\alpha_b(t_2))\right],$$

using (2.10) and (2.11) and let $t_1-t_2=v,t_2=t$, and using assumption I. then we have, $\gamma_{ab}^{(T)}(0)=O(T)$, and

$$E\left[I_{ab}^{(T)}(h)\right]=P_{ab}\begin{bmatrix}f_{a_1a_2}(h) & f_{a_1b_2}(h)\\ f_{b_1a_2}(h) & B(h)f_{a_1a_2}(h)B(h)^T\end{bmatrix}+$$
$$+P_{ab}O(T^{-1})\begin{bmatrix}O(1) & O(1)\\ O(1) & O(1)\end{bmatrix}$$

$$= P_{ab}\begin{bmatrix} f_{a_1a_2}(h) & f_{a_1b_2}(h) \\ f_{b_1a_2}(h) & B(h) f_{a_1a_2}(h) B(h)^T \end{bmatrix} + \begin{bmatrix} O(T^{-1}) & O(T^{-1}) \\ O(T^{-1}) & O(T^{-1}) \end{bmatrix},$$

and,

$$Cov\left[I^{(T)}_{a_1b_1}(h), I^{(T)}_{a_2b_2}(\hbar)\right] = \left\{2\pi\gamma^{(T)}_{a_1b_1}(0)\right\}^{-1}\left\{2\pi\gamma^{(T)}_{a_2b_2}(0)\right\}^{-1}$$

$$Cov\left\{C^{(T)}_{a_1}(h)C^{(T)}_{b_1}(-h), C^{(T)}_{a_2}(\hbar)C^{(T)}_{b_2}(-\hbar)\right\}$$

$$= (2\pi)^{-2}\left\{\gamma^{(T)}_{a_1b_1}(0)\gamma^{(T)}_{a_2b_2}(0)\right\}^{-1} \times$$

$$\times \left\{Cov\left\{C^{(T)}_{a_1}(h), C^{(T)}_{a_2}(\hbar)\right\}Cov\left\{C^{(T)}_{b_1}(-h), C^{(T)}_{b_2}(-\hbar)\right\} +\right.$$

$$+ \left\{Cov\left\{C^{(T)}_{a_1}(h), C^{(T)}_{b_2}(-\hbar)\right\}Cov\left\{C^{(T)}_{b_1}(-h), C^{(T)}_{a_2}(\hbar)\right\} +\right.$$

$$\left. + cum\left\{C^{(T)}_{a_1}(h), C^{(T)}_{b_1}(-h), C^{(T)}_{a_2}(\hbar), C^{(T)}_{b_2}(-\hbar)\right\}\right\}$$

$$= D(V + N + S)$$

where,

$$D = (2\pi)^{-2}\left\{\gamma^{(T)}_{a_1b_1}(0)\gamma^{(T)}_{a_2b_2}(0)\right\}^{-1},$$

$$V = \left\{Cov\left\{C^{(T)}_{a_1}(h), C^{(T)}_{a_2}(\hbar)\right\}Cov\left\{C^{(T)}_{b_1}(-h), C^{(T)}_{b_2}(-\hbar)\right\}\right\},$$

$$N = \left\{Cov\left\{C^{(T)}_{a_1}(h), C^{(T)}_{a_2}(-\hbar)\right\}Cov\left\{C^{(T)}_{b_1}(-h), C^{(T)}_{b_2}(\hbar)\right\}\right\},$$

$$S = cum\left\{C^{(T)}_{a_1}(h), C^{(T)}_{b_1}(-h), C^{(T)}_{a_2}(\hbar), C^{(T)}_{b_2}(-\hbar)\right\}.$$

Now,

$$V = \left\{Cov\left\{C^{(T)}_{a_1}(h), C^{(T)}_{a_2}(\hbar)\right\}Cov\left\{C^{(T)}_{b_1}(-h), C^{(T)}_{b_2}(-\hbar)\right\}\right\}$$

$$V = V_1 \times V_2$$

$$V_1 = Cov\left\{C^{(T)}_{a_1}(h), C^{(T)}_{a_2}(\hbar)\right\}$$

$$= \sum_{t_1,t_2=0}^{T-1} \lambda^{(T)}_{a_1}(t_1)\lambda^{(T)}_{a_2}(t_2)\exp(-iht_1 + i\hbar t_2) \times$$

$$\times \begin{bmatrix} p_{a_1a_2}R_{xx}(t_1 - t_2) & p_{a_1a_2}R_{x\psi}(t_1 - t_2) \\ p_{a_1a_2}R_{\psi x}(t_1 - t_2) & p_{a_1a_2}B(h)R_{xx}(t_1 - t_2)B(h)^T \end{bmatrix},$$

putting $t_1 - t_2 = v_1, t_2 = t \Rightarrow t_1 = t + v_1$, and using assumption I. Then we have

$$V_1 = 2\pi p_{a_1a_2}\gamma_{a_1a_2}(h - \hbar)\begin{bmatrix} f_{a_1a_2}(h) & f_{a_1b_2}(h) \\ f_{b_1a_2}(h) & B(h) f_{a_1a_2}(h) B(h)^T \end{bmatrix} + \begin{bmatrix} O(1) & O(1) \\ O(1) & O(1) \end{bmatrix}, \quad (2.15)$$

$$V_2 = Cov\left\{C^{(T)}_{b_1}(-h), C^{(T)}_{b_2}(-\hbar)\right\}$$

$$= Cov\left\{\begin{aligned} &\sum_{t_1=0}^{T-1}\lambda^{(T)}_{b_1}(t_1)\alpha_{b_1}(t_1)\exp(iht_1), \\ &\sum_{t_2=0}^{T-1}\lambda^{(T)}_{b_2}(t_2)\alpha_{b_2}(t_2)\exp(i\hbar t_2) \end{aligned}\right\}$$

$$= \sum_{t_1,t_2=0}^{T-1} \lambda^{(T)}_{b_1}(t_1)\lambda^{(T)}_{b_2}(t_2)\exp(iht_1 - i\hbar t_2) \times$$

$$\times \begin{bmatrix} p_{b_1b_2}R_{xx}(t_1 - t_2) & p_{b_1b_2}R_{x\psi}(t_1 - t_2) \\ p_{b_1b_2}R_{\psi x}(t_1 - t_2) & p_{b_1b_2}B(h)R_{xx}(t_1 - t_2)B(h)^T \end{bmatrix},$$

putting $t_1 - t_2 = v_1, t_2 = t \Rightarrow t_1 = t + v_1$. Then we have

$$V_2 = 2\pi p_{b_1b_2}\gamma_{b_1b_2}(-h + \hbar) \times$$

$$\times \begin{bmatrix} f_{a_1a_2}(-h) & f_{a_1b_2}(-h) \\ f_{b_1a_2}(-h) & B(h) f_{a_1a_2}(-h) B(h)^T \end{bmatrix} + \begin{bmatrix} O(1) & O(1) \\ O(1) & O(1) \end{bmatrix}, \quad (2.16)$$

$$N = \left\{Cov\left\{C^{(T)}_{a_1}(h), C^{(T)}_{a_2}(-\hbar)\right\}Cov\left\{C^{(T)}_{b_1}(-h), C^{(T)}_{b_2}(\hbar)\right\}\right\}$$

$$N = N_1 \times N_2$$

$$N_1 = Cov\left\{C^{(T)}_{a_1}(h), C^{(T)}_{b_2}(-\hbar)\right\}$$

$$N_1 = Cov\left\{\begin{aligned} &\sum_{t_1=0}^{T-1}\lambda^{(T)}_{a_1}(t_1)\alpha_{a_1}(t_1)\exp(-iht_1), \\ &\sum_{t_2=0}^{T-1}\lambda^{(T)}_{b_2}(t_2)\alpha_{b_2}(t_2)\exp(i\hbar t_2) \end{aligned}\right\}$$

$$= \sum_{t_1,t_2=0}^{T-1} \lambda^{(T)}_{a_1}(t_1)\lambda^{(T)}_{b_2}(t_2)\exp(-iht_1 - i\hbar t_2) \times$$

$$\times \begin{bmatrix} p_{a_1b_2}R_{xx}(t_1 - t_2) & p_{a_1b_2}R_{x\psi}(t_1 - t_2) \\ p_{a_1b_2}R_{\psi x}(t_1 - t_2) & p_{a_1b_2}B(h)R_{xx}(t_1 - t_2)B(h)^T \end{bmatrix},$$

putting $t_1 - t_2 = v_1, t_2 = t \Rightarrow t_1 = t + v_1$. Then we have

$$N_1 = 2\pi\, p_{a_1b_2}\gamma_{a_1b_2}(h + \hbar) \times$$

$$\times \begin{bmatrix} f_{a_1a_2}(h) & f_{a_1b_2}(h) \\ f_{b_1a_2}(h) & B(h) f_{a_1a_2}(h) B(h)^T \end{bmatrix} + \begin{bmatrix} O(1) & O(1) \\ O(1) & O(1) \end{bmatrix}, \quad (2.17)$$

$$N_2 = Cov\left\{C^{(T)}_{b_1}(-h), C^{(T)}_{a_2}(\hbar)\right\}$$

$$N_2 = 2\pi p_{b_1a_2}\gamma_{b_1a_2}(-h-\hbar)\begin{bmatrix} f_{a_1a_2}(-h) & f_{a_1b_2}(-h) \\ f_{b_1a_2}(-h) & B(h)f_{a_1a_2}(-h)B(h)^T \end{bmatrix}+$$

$$+\begin{bmatrix} O(1) & O(1) \\ O(1) & O(1) \end{bmatrix}, \qquad (2.18)$$

$$S = cum\left\{C_{a_1}^{(T)}(h), C_{b_1}^{(T)}(-h), C_{a_2}^{(T)}(\hbar), C_{b_2}^{(T)}(-\hbar)\right\}$$

$$= \sum_{t_1,t_2,t_3,t_4=0}^{T-1} \lambda_{a_1}^{(T)}(t_1)\lambda_{b_1}^{(T)}(t_2)\lambda_{a_2}^{(T)}(t_3)\lambda_{b_2}^{(T)}(t_4)\exp(-iht_1)\times$$

$$\times \exp(iht_2)\exp(-i\hbar t_3)\exp(i\hbar t_4)\times$$

$$\times Cov\left\{\alpha_{a_1}(t_1), \alpha_{b_1}(t_2), \alpha_{a_2}(t_3), \alpha_{b_2}(t_4)\right\}$$

$$= p_{a_1b_1a_2b_2}(2\pi)^3 f_{\alpha_{a_1}\alpha_{b_1}\alpha_{a_2}\alpha_{b_2}}(h,-h,\hbar)\gamma_{a_1b_1a_2b_2}{}^{(T)}(0) + O(1)$$

where,

$$\gamma_{a_1b_1a_2b_2}^{(T)}(0) = \sum_{t=0}^{T-1} \lambda_{a_1}^{(T)}(t+v_1)\lambda_{b_1}^{(T)}(t+v_2)\lambda_{a_2}^{(T)}(t+v_3)\lambda_{b_2}^{(T)}(t),$$

now,

$$Cov\left[I_{a_1b_1}^{(T)}(h), I_{a_2b_2}^{(T)}(\hbar)\right] = D(V+N+S)$$

$$= D\left\{(V_1\times V_2) + (N_1\times N_2) + S\right\}$$

$$= (2\pi)^{-2}\left\{\gamma_{a_1b_1}^{(T)}(0)\gamma_{a_2b_2}^{(T)}(0)\right\}^{-1}\times$$

$$\times\left\{(2\pi\, p_{a_1a_2}\gamma_{a_1a_2}(h-\hbar)\times\eta + \mathrm{O})\times\right.$$

$$\times(2\pi\, p_{b_1b_2}\gamma_{b_1b_2}(-h+\hbar)\times A + \mathrm{O}) +$$

$$+(2\pi\, p_{a_1b_2}\gamma_{a_1b_2}(h+\hbar)\times\eta + \mathrm{O})\times$$

$$\times(2\pi\, p_{b_1a_2}\gamma_{b_1a_2}(-h-\hbar)\times A + \mathrm{O}) +$$

$$\left.+\left(p_{a_1b_1a_2b_2}(2\pi)^3 f_{\alpha_{a_1}\alpha_{b_1}\alpha_{a_2}\alpha_{b_2}}(h,-h,\hbar)\gamma_{a_1b_1a_2b_2}^{(T)}(0) + O(1)\right)\right\},$$

where,

$$\eta = \begin{bmatrix} f_{a_1a_2}(h) & f_{a_1b_2}(h) \\ f_{b_1a_2}(h) & B(h)f_{a_1a_2}(h)B(h)^T \end{bmatrix},$$

$$A = \begin{bmatrix} f_{a_1a_2}(-h) & f_{a_1b_2}(-h) \\ f_{b_1a_2}(-h) & B(h)f_{a_1a_2}(-h)B(h)^T \end{bmatrix},$$

$$\mathrm{O} = \begin{bmatrix} O(1) & O(1) \\ O(1) & O(1) \end{bmatrix},$$

then,

$$Cov\left[I_{a_1b_1}^{(T)}(h), I_{a_2b_2}^{(T)}(\hbar)\right] = (2\pi)^{-2}\left\{\gamma_{a_1b_1}^{(T)}(0)\gamma_{a_2b_2}^{(T)}(0)\right\}^{-1}\times$$

$$\times\left[(2\pi)^2 p_{a_1a_2}p_{b_1b_2}\gamma_{a_1a_2}(h-\hbar)\gamma_{b_1b_2}(-h+\hbar)\eta A + \right.$$

$$\left.+(2\pi)^2 p_{a_1b_2}p_{b_1a_2}\gamma_{a_1b_2}(h+\hbar)\gamma_{b_1a_2}(-h-\hbar)\eta A\right] +$$

$$+(2\pi)^{-2}\left\{\gamma_{a_1b_1}^{(T)}(0)\gamma_{a_2b_2}^{(T)}(0)\right\}^{-1}\times O\times$$

$$\times\left[2\pi\, p_{a_1a_2}\gamma_{a_1a_2}(h-\hbar)A + 2\pi\, p_{b_1b_2}\gamma_{b_1b_2}(-h+\hbar)\eta + \right.$$

$$\left.+2\pi\, p_{a_1a_2}\gamma_{a_1a_2}(h-\hbar)A + 2\pi\, p_{b_1b_2}\gamma_{b_1b_2}(-h+\hbar)\eta\right] +$$

$$+(2\pi)^{-2}\left\{\gamma_{a_1b_1}^{(T)}(0)\gamma_{a_2b_2}^{(T)}(0)\right\}^{-1}\times$$

$$\times\left[(2\pi)^3 p_{a_1b_1a_2b_2} f_{\alpha_{a_1}\alpha_{b_1}\alpha_{a_2}\alpha_{b_2}}(h,-h,\hbar)\gamma_{a_1b_1a_2b_2}^{(T)}(0) + O(1)\right] +$$

$$+(2\pi)^{-2}\left\{\gamma_{a_1b_1}^{(T)}(0)\gamma_{a_2b_2}^{(T)}(0)\right\}^{-1}\left[O^2 + O^2\right],$$

$$= K_1 + K_2 + K_3 + K_4 \qquad (2.19)$$

From the bounded of $\lambda_a^{(T)}(t)$, we have $\gamma_{ab}^{(T)}(0) = O(T)$ and $\gamma_{abcd}^{(T)}(0) = O(T)$, $1,\ldots,\min(i,j)$, and $f_{\alpha_{a_1}\alpha_{b_1}\alpha_{a_2}\alpha_{b_2}}(h,-h,\hbar)$ is bounded by a constant H, $a_i, b_i = 1,\ldots,\min(i,j)$, $i = 1,\ldots,k$, $h,\hbar \in R$, then,

$$K_3 = O(T^{-1}) + O(T^{-2}) = O(T^{-1})$$

Also,

$$K_4 = (2\pi)^{-2}\left\{\gamma_{a_1b_1}^{(T)}(0)\gamma_{a_2b_2}^{(T)}(0)\right\}^{-1}\left[O^2 + O^2\right]$$

$$= (2\pi)^{-2}\left\{\gamma_{a_1b_1}^{(T)}(0)\gamma_{a_2b_2}^{(T)}(0)\right\}^{-1}\times O$$

$$= (2\pi)^{-2}\left\{O(T)O(T)\right\}^{-1}\times O$$

$$= O(T^{-2})\times O = O(T^{-2})\begin{bmatrix} O(1) & O(1) \\ O(1) & O(1) \end{bmatrix} =$$

$$= \begin{bmatrix} O(T^{-2}) & O(T^{-2}) \\ O(T^{-2}) & O(T^{-2}) \end{bmatrix},$$

using K_3, K_4 into (2.19). Then,

$$Cov\left[I_{a_1b_1}^{(T)}(h), I_{a_2b_2}^{(T)}(\hbar)\right] = \left\{\gamma_{a_1b_1}^{(T)}(0)\gamma_{a_2b_2}^{(T)}(0)\right\}^{-1}\times$$

$$\times\left[(p_{a_1a_2}p_{b_1b_2}\gamma_{a_1a_2}(h-\hbar)\overline{\gamma_{b_1b_2}(h-\hbar)}\eta A\right.$$

$$\left.+ p_{a_1b_2}p_{b_1a_2}\gamma_{a_1b_2}(h+\hbar)\overline{\gamma_{b_1a_2}(h+\hbar)}\eta A\right]$$

$$+T^{-2}N_{a_1b_1a_2b_2}^{(T)}{}^{(T)}(h,\hbar) + O(T^{-1}),$$

where there exists any constant N such that,

$$\left|T^{-2} N_{a_1 b_1 a_2 b_2}^{(T)} {}^{(T)}(h,\hbar)\right| \leq$$
$$W\left\{\left|\gamma_{a_1 a_2}(h-\hbar)\right| + \left|\overline{\gamma_{b_1 b_2}(h-\hbar)}\right| + \left|\gamma_{a_1 b_2}(h+\hbar)\right| + \left|\overline{\gamma_{b_1 a_2}(h+\hbar)}\right|\right\}$$

Then the proof is obtained.

3 The Dispersion For The Modified Periodogram For Two Vector Valued Stationary Time Series With Missed Observations.

In this section we will study the dispersion for the modified periodogram for two vector valued stationary time series with missed observations by the following corollaries.

Corollary 3.1. *Suppose that* $\alpha_a(t) = \ell_a(t)\tau_a(t)$, $a = 1,2,..,$ $\min(i, j)$ *are missed observations on the strictly stationary discrete stochastic processes which satisfies assumption II with mean zero, let* $\lambda_a(t), -\infty < t < \infty$ *, and let*

$$I_{\alpha\alpha}^{(T)}(h) = \left[I_{ab}^{(T)}(h)\right] = \left[\left\{2\pi\gamma_{ab}^{(T)}(0)\right\}^{-1} C_a^{(T)}(h)\overline{C_b^{(T)}(h)}\right],$$

then,

$$E\left[I_{ab}^{(T)}(h)\right] \to p_{a_1 a_2}\begin{bmatrix} f_{a_1 a_2}(h) & f_{a_1 b_2}(h) \\ f_{b_1 a_2}(h) & B(h) f_{a_1 a_2}(h) B(h)^T \end{bmatrix}$$

as $T \to \infty$*,* $a, b = 1, \ldots, \min(i, j)$*,* $h \in R$.

Proof.

From (2.13) and by taking the limits for both sides then the proof comes directly by using the given constraints.

In Corollary (2.2) below we use of the Kroncker delta function which is given by

$$\Theta(h) = \begin{cases} 1 \;\;, \; if \;\; h = 0 \\ 0 \;\;, \; o.w \end{cases} , \qquad (3.1)$$

which is dependence of $I_{a_1 b_1}^{(T)}(h)$ and $I_{a_2 b_2}^{(T)}(\hbar)$, $a_i, b_i =$ $1,2,\ldots,\min(i, j)$, $i = 1,\ldots,k, h, \hbar \in R$ is seen to fall off as the function $\gamma_{ab}{}^{(T)}(h)$, $a, b = 1,2,\ldots,\min(r, s)$, $h \in R$ fall off. By using the limit, theorem (2.2) becomes:

Now, when $h \pm \hbar \neq 0$, $h, \hbar \in R$, for some constants F, and the bounded of $f_{ab}(h)$ $a, b = 1,2, \ldots,$ $\min(i, j)$, $h \in R$ then taking the modulus for both sides of (2.19) and using theorem (2.1), we have At $h \pm \hbar = 0$, using assumption (I), then we get from (2.14), and using the limit, then,

Corollary 3.2. *For all* $h, \hbar \in R$ *and under the constraints of theorem (2.2) then,*

$$Cov\left[I_{a_1 b_1}^{(T)}(h), I_{a_2 b_2}^{(T)}(\hbar)\right] = P^4 \Theta(h-\hbar)\eta A +$$
$$+ P^4 \Theta(h+\hbar) B \eta A + O(T^{-1}),$$

$$\lim_{T\to\infty} Cov\left[I_{a_1 b_1}^{(T)}(h), I_{a_2 b_2}^{(T)}(\hbar)\right] =$$
$$= \begin{cases} P^4\Theta(h-\hbar)\eta A + P^4\Theta(h+\hbar)\eta A & if \;\; h \pm \hbar = 0 \\ 0 & if \;\; h \pm \hbar \neq 0 \end{cases}$$

Proof.

$$Cov\left[I_{a_1 b_1}^{(T)}(h), I_{a_2 b_2}^{(T)}(\hbar)\right] = P^4\Theta(h-\hbar)\eta A + P^4\Theta(h+\hbar)\eta A$$

$$\left|Cov\left[I_{a_1 b_1}^{(T)}(h), I_{a_2 b_2}^{(T)}(\hbar)\right]\right| \leq \left\{\gamma_{a_1 b_1}^{(T)}(0)\gamma_{a_2 b_2}^{(T)}(0)\right\}^{-1} \times$$
$$\times \left\{\left[\frac{2\Gamma_1 u_1}{\left|\sin(h+\hbar)/2\right|}\right]^2 F^2 + \left[\frac{2\Gamma_2 u_2}{\left|\sin(h-\hbar)/2\right|}\right]^2 F^2\right\} +$$
$$+ T^{-2}\left|T^{-2} N_{a_1 b_1 a_2 b_2}^{(T)}(h,\hbar)\right| + (T^{-1})$$

by using corollary (2.1) we get $Cov\left[I_{a_1 b_1}^{(T)}(h), I_{a_2 b_2}^{(T)}(\hbar)\right] \to 0$ as $T \to \infty$, the proof is obtained.

In the case of $h \pm \hbar = 0$ then the previous corollary indicates the following one.

Corollary 3.3 *Using theorem (2.2) and corollary (3.2) then we have,*

$$\lim_{T\to\infty} D[I_{ab}^{(T)}(h)] =$$
$$= \begin{cases} P^4\Theta(h-\hbar)\eta A & if \;\; h = \hbar = \zeta \neq 0 \\ P^4\Theta(h-\hbar)\eta A + P^4\Theta(h+\hbar)\eta A & if \;\; h = \hbar = \zeta = 0 \end{cases}$$

Proof.

Let $h = \hbar = \zeta$, $\zeta \in R$, $a_1 = a_2 = a$, $b_1 = b_2 = b$, $a, b = 1,2,\ldots,\min(i, j)$ into corollary (3.2), we get

$$\lim_{T\to\infty} D[I_{ab}^{(T)}(h)] = P^4\Theta(\zeta-\zeta)\eta A + P^4\Theta(\zeta+\zeta)\eta A$$

when $\zeta = 0$, then, $\lim_{T\to\infty} D[I_{ab}^{(T)}(h)] = P^4\eta A + P^4\eta A$.

Hence the proof is obtained.

4 Conclusion

It is clear from the study that the properties of the modified periodogram for stationary time series of two vector valued with missed observations are approximately the same properties to the classical one, which will lead to apply in many important fields such as economy, astronomy, and medicine.

ISSN 2229-5518

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